Calculus III: Final Exam

Fall 2014

Name: ____ N____________________

This exam is scheduled for 110 minutes. No calculators, notes, or other outside materials are permitted. **Show all work to receive full credit, except where specified.** The exam is worth 80 points.

Please mark an "X" next to your section and write in your recitation section number.

X	Sect.	Instructor	Time	Recitation
	001	Cerfon	Mon, Wed 8:55 AM – 10:45 AM in Silv 410	
	002	Lai	Tue, Thu 3:30 PM – 5:20 PM in GCASL 261	
	003	Ristroph	Tue, Thu 8:55 AM – 10:45 AM in GCASL 275	
	004	Dies	Tue, Thu 6:20 PM – 8:10 PM in 25W 4th C-8	
	005	Leingang	Tue, Thu 8:55 AM – 10:45 AM in Silv 410	
	006	Kallemov	Mon, Wed 11:00AM – 12:50PM in 7E12 125	
	007	Youngren	Tue, Thu 11:00AM – 12:50PM in Wave 366	
	008	Trogdon	Mon, Wed 3:30PM – 5:20PM in 7E12 125	

Name: ______________________________

Multiple Choice Answer Sheet

Be sure to put all your final answers here as only this sheet will be graded. **DO NOT DETACH.**

1	A	B	C	D	E
2	A	B	C	D	E
3	A	B	C	D	E
4	A	B	C	D	E
5	A	B	C	D	E
6	A	B	C	D	E
7	A	B	C	D	E
8	A	B	C	D	E
9	A	B	C	D	E
10	A	B	C	D	E

11	T	F
12	T	F
13	T	F
14	T	F
15	T	F

For scoring purposes only. Do not mark below this line.

MC	FR1	FR2	FR3	FR4	Total
30	15	15	15	15	90

Multiple Choice

Be sure your final answers to each multiple choice question are marked on the accompanying answer sheet.

(2 points each) Select the correct answer for each question. *You need not justify your answer. No partial credit will be given.*

MC1 Which of the following vectors is **parallel** to the plane $x-2y+3z=6$?

- (A) $\langle 1,2,3\rangle$
- (B) $\langle -1,2,-3\rangle$
- (C) $\langle 1,2,1\rangle$
- (D) $\langle 1,-1,1\rangle$
- (E) none of these

MC2 Find an equivalent expression to the iterated integral

$$\int_1^{e^2}\int_0^{\ln x} f(x,y)\,dy\,dx$$

- (A) $\int_0^2\int_1^{e^2} f(x,y)\,dx\,dy$
- (B) $\int_1^{e^2}\int_0^{\ln x} f(y,x)\,dy\,dx$
- (C) $\int_0^2\int_0^{e^y} f(x,y)\,dx\,dy$
- (D) $\int_0^2\int_{e^y}^{e^2} f(x,y)\,dx\,dy$
- (E) $\int_1^{e^2}\int_0^{\ln\theta} f(r,\theta)\,r\,dr\,d\theta$

MC3 Find values for constants a and b such that the function

$$f(x,y)=x^2+ax+by^2-3$$

has a local maximum at the point $(1,0)$.

- (A) $a=-2,\quad b=0$
- (B) $a=0,\quad b=1$
- (C) $a=0,\quad b=-1$
- (D) $a=-2,\quad b=-1$
- (E) No such values are possible.

MC4 Let E be the region in the first octant of $\mathbb{R}^3$ contained in the sphere $x^2+y^2+z^2=4$.

Which iterated integral below computes the triple integral $\iiint_E z\,dV$?

(A) $\int_0^2\int_0^2\int_0^2 \sqrt{4-x^2-y^2-z^2}\,dx\,dy\,dz$

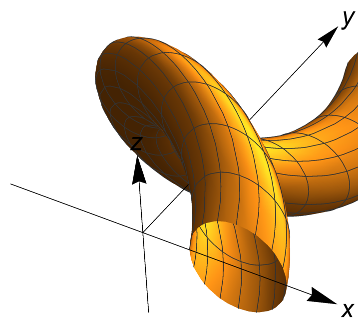
(B) $\int_0^4\int_0^{\pi}\int_0^{\pi/2} \rho\cos\phi\,d\phi\,d\theta\,d\rho$

(C) $\int_0^2\int_0^{\pi}\int_0^2 \sqrt{4-r^2}\,r\,dz\,d\theta\,dr$

(D) $\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^3\cos\phi\sin\phi\,d\rho\,d\theta\,d\phi$

(E) All of these.

MC5 Match the surface below to its parameterization.



$\mathbf{r}(u,v) =$

(A) $\langle (1+v)\cos u, 1-u, (1+v)\sin u\rangle$

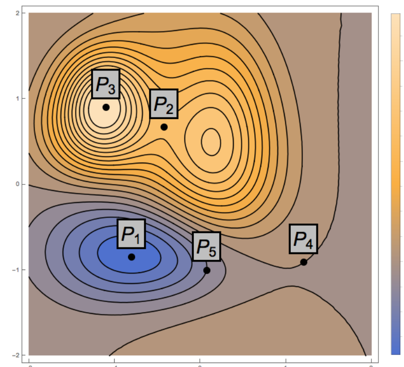
(B) $\langle \sin v\cos u, \sin v\sin u, u^2 v\rangle$

(C) $\langle \sin u\cos v, u^3-u+4, \cos u\sin v\rangle$

(D) $\langle (2-\sin(\pi u))\cos v, u, (2-\sin(\pi u))\sin v\rangle$

(E) $\langle \cos(v)+2\cos(u), u, \sin(v)+2\sin(u)\rangle$

MC6 Which marked point in the contour plot below is the most likely location of a saddle point?



(A) P_4

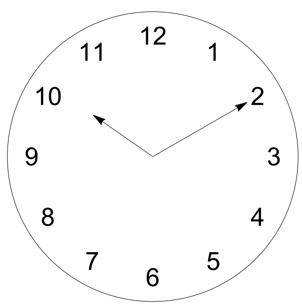
(B) P_5

(C) P_2

(D) P_3

(E) P_1

MC7 Consider the hands of a typical clock face as representing vectors in $\mathbb{R}^2$. At which time is their dot product **greatest**?



(A) 11:50

(B) 12:15

(C) 9:30

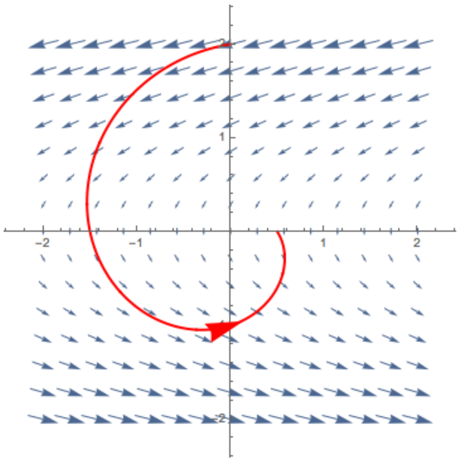
(D) 6:00

(E) 3:15

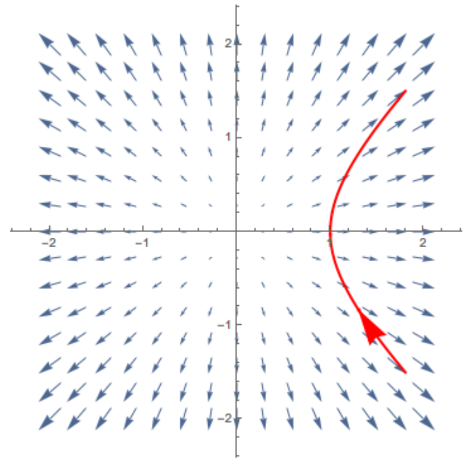
MC8 Each graph below represents a vector field **F** and a piecewise smooth curve C. Select the one with the **largest** value for

$$\oint_C \mathbf{F} \cdot d\mathbf{r}.$$

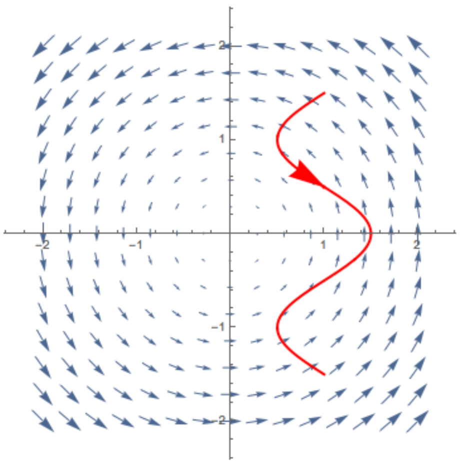
A



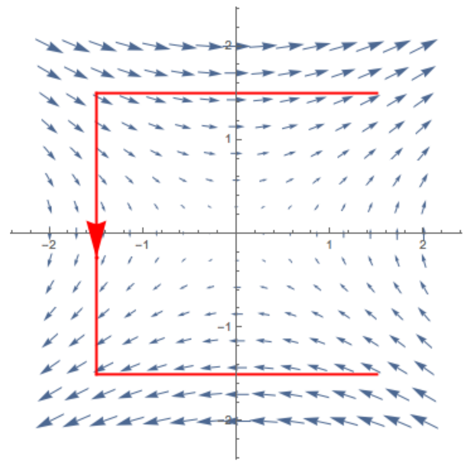
D



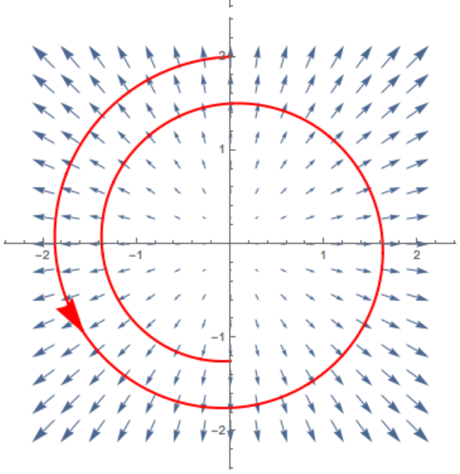
B



E



C



MC9 Let f, g be smooth, scalar valued functions on $\mathbb{R}^3$ and $\mathbf{F}, \mathbf{G}$ smooth vector fields on $\mathbb{R}^3$. Which of the following is an **invalid** expression?

- (A) $\nabla \times \mathbf{F}$
- (B) $\nabla f \times \mathbf{F}$
- (C) $\nabla \cdot \nabla f$
- (D) $(\nabla \cdot \mathbf{F})\mathbf{G}$
- (E) $\nabla \times \nabla \cdot \mathbf{F}$

MC10 Let $\mathbf{r}(t) = 3t^2\,\mathbf{i} + 4t\,\mathbf{j} + 5\,\mathbf{k}$ represent the position of a particle at time t. Which of the following is the component (scalar projection) of the velocity vector of the particle in the direction $\mathbf{w} = 2\,\mathbf{i} - \mathbf{j} + 2\,\mathbf{k}$ at time $t = 1$?

- (A) 4
- (B) $\frac{8}{3}$
- (C) $\frac{8}{9}$
- (D) $\sqrt{52}$
- (E) $\frac{\sqrt{52}}{3}$

MC11 The surface integral with respect to surface area

$$\iint_{\mathcal{S}} f(x,y,z)\,dS$$

is only defined for orientable surfaces $\mathcal{S}$.

Ⓣ True Ⓕ False

MC12 If a function $f(x,y)$ is differentiable at the origin, then $D_{\mathbf{u}}f(0,0)$ must have a minimum value for some direction $\mathbf{u}$.

Ⓣ True Ⓕ False

MC13 There exist nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ such that

$$\mathbf{u}\cdot\mathbf{v} = |\mathbf{u}\times\mathbf{v}|.$$

Ⓣ True Ⓕ False

MC14 Every smooth vector field is the curl of some other vector field.

Ⓣ True Ⓕ False

MC15 The curl of a conservative vector field is always $\mathbf{0}$.

Ⓣ True Ⓕ False

STOP! MAKE SURE YOUR FINAL ANSWERS ARE MARKED ON THE SHEET INSIDE THE FRONT COVER.

Free Response

Be sure to show all your work neatly and indicate your final answer where appropriate.

FR1 (15 points)

(a) (7 points) Let L be the line given by $\mathbf{r}(t) = -2\,\mathbf{i} + t\,\mathbf{j} - t\,\mathbf{k}$. Find the line containing the point $(1,4,2)$ and perpendicular to L (lines must intersect to be considered perpendicular).

(b) (8 points) Use the method of Lagrange multipliers to find the point on the line $2x+3y=6$ closest to the origin.

FR2 (15 points) Let C be a piecewise smooth curve from $(0,0)$ to $(\pi/2,0)$. Evaluate the following line integral

$$\int_C \cos x \cos y \, dx + (1 - \sin x \sin y) \, dy$$

two ways:

(a) (8 points) by using the Fundamental Theorem of Line Integrals.

(b) (7 points) by directly computing the integral along a convenient path.

FR3 (15 points) Let $\mathbf{F} = -z^2\,\mathbf{i} + xy\,\mathbf{j} + 2x\,\mathbf{k}$ and let C be the triangle with vertices at $(0,0,0)$, $(3,3,0)$, and $(0,0,6)$ (oriented in that order).

Use Stokes' Theorem to compute the line integral

$$\oint_C \mathbf{F} \cdot d\mathbf{r}.$$

FR4 (15 points) Let S be the upper hemisphere described by $x^2+y^2+z^2=16$ with $z \geq 0$ (oriented upward). We will use the Divergence Theorem to compute the flux of the vector field

$$\mathbf{F}(x,y,z) = (xz - yx + 2x)\,\mathbf{i} + (2y^2 - 2xy - yz)\,\mathbf{j} + (2zx - 3zy)\,\mathbf{k}$$

through S as follows.

(a) (3 points) Let D be the part of the xy-plane under the hemisphere. Find $\mathbf{n}$, the outward-pointing normal vector to D.

(b) (3 point) Compute $\mathbf{F} \cdot \mathbf{n}$ on D.

(c) (3 points) Compute $\operatorname{div} \mathbf{F}$.

(d) (6 points) Use the Divergence Theorem to complete the computation.