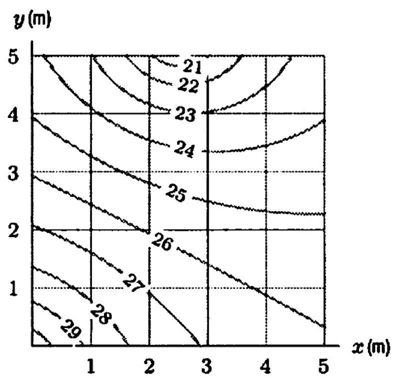
New York University
MATH.UA 123 Calculus 3

Problem Set 4

This problem set consists not only of problems similar to what you've seen, but also of unique problems you may not have seen before. The purpose of the latter is for you to apply the concepts you've previously learned to new, unfamiliar, and usually more interesting situations. In some cases, problems connect ideas from multiple learning objectives.

Write full, clear solutions to the problems below. It is important that the logic of how you solved these problems is clear. Although the final answer is important, being able to convey you understand the underlying concepts is more important. The point weight of each problem is indicated prior to each question. This problem set is graded out of 50 total points.

1. (5 points) The figure below shows the distribution of temperature, in °C, in a 5 meter by 5 meter heated room. Using Riemann sums, estimate the average temperature in the room.



2. (5 points) Sketch the solid that lies between the surface $z = \frac{2xy}{x^2+1}$ and the plane $z = x + 2y$ and is bounded by the planes $x = 0$, $x = 2$, $y = 0$, and $y = 4$. Then, find its volume.

3. (5 points) Evaluate the double integral
$$\int\int_R \frac{y}{x^2y^2+1}\, dA,$$
over the region R: $0 \le x \le 1$, $-1 \le y \le 2$.

4. (5 points) Sketch the region of integration and evaluate the integral:
$$\int_1^4 \int_{\sqrt{y}}^{y} x^2y^3\, dx\, dy.$$

5. (5 points) (a) Sketch the region in the xy-plane that is bounded by the x-axis, $y = x$, and $x + y = 2$.
   (b) Express the integral of $f(x, y)$ over this region in terms of iterated integrals in two ways. (That is, formulate the integral in two ways: in one, use $dx\, dy$; in the other, use $dy\, dx$.)
   (c) Using one of your answers to part (b), evaluate the integral exactly for $f(x, y) = x$.

6. (5 points) If R is the region $x + y \ge a$, $x^2 + y^2 \le a^2$, with $a > 0$, evaluate the integral
$$\int_R xy\, dA.$$

7. (5 points) (a) Sketch the level curves of the function $f(x, y) = 4 - x^2 - 2y^2$, at levels $k = 4, 3, 0$, and -5.

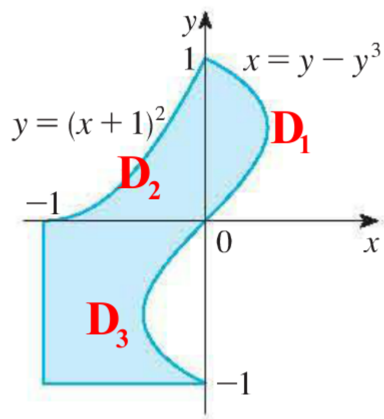
   (b) What region R in the xy-plane maximizes the value of

   $$\int\int_R (4 - x^2 - 2y^2)\ dA\ ?$$

   Give reasons for your answer.

   (c) Then, express the double integral over the region R you specified above as an iterated integral.

8. (5 points) Express D, the shaded region below, as a union of regions of type I or type II and evaluate the integral $\displaystyle\int\int_D y\ dA$.



9. (5 points) (a) Sketch the region of integration of

   $$\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{4-x^2}} x\ dy\ dx + \int_1^2 \int_0^{\sqrt{4-x^2}} x\ dy\ dx.$$

   (b) Evaluate the integral in part (a) by first converting into polar coordinates.

10. (5 points) Find the volume of an ice cream cone bounded by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and the cone $z = \sqrt{x^2 + y^2}$.