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Written assignment #1

Problem 1. The Babylonian algorithm for computing $y = \sqrt{x}$ works by first choosing $y_0 > 0$ and then iterating:

$$y_{k+1} \leftarrow \frac{1}{2}\left(\frac{x}{y_k} + y_k\right) \tag{1}$$

until $|y_{k+1} - y_k|$ is below some tolerance. This is a fixed point iteration of the form $y_{k+1} = f(y_k)$, where $f = (x/y_k + y_k)/2$.

If $|y_{k+1} - y_k|$ becomes small, then the iteration has converged. But this alone is insufficient to conclude that y_k has converged to $\sqrt{x}$. However, we do know that $\sqrt{x}$ is a fixed point of f (i.e., $\sqrt{x} = f(\sqrt{x})$).

Use the contraction mapping theorem to prove that if $y_0 > 0$, the Babylonian algorithm converges to $\sqrt{x}$. ***Hint***: Let $a < b$ define a closed and bounded interval $[a, b]$. What can you say about $f([a, b])$? That is, what does the interval $[a, b]$ map to under f? Can you choose a and b independently, or does one need to depend on the other?