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Written assignment #2

Problem 1. Let $a, b \in \mathbb{R}$, with $b > 0$. An algorithm for division (i.e., for computing a/b) on early computers is based on the following idea:

> First, compute $b^{-1} = 1/b$ by applying Newton's method to the function $f(x) = b - 1/x$. Afterwards, form the product $a/b = a \cdot b^{-1}$.

In this problem we'll work out the details of this idea:

1. Show that the Newton iteration is equivalent to the iteration:

$$x_{n+1} = x_n(2 - bx_n), \qquad n \geq 0. \tag{1}$$

2. Prove that this iteration converges if and only if $0 < x_0 < 2/b$.
3. Make a plot (using `matplotlib`) and give an explanation which shows why this condition makes sense.
4. Implement this iteration and use it to compute $b^{-1} = 1/3$.

Problem 2. Let f be a twice continuously differentiable function ($f \in C^2$). We can assume that f is C^2 on all of $\mathbb{R}$ for simplicity. Let $\xi \in \mathbb{R}$ such that $f(\xi) = f'(\xi) = 0$. That is, f has a *double root* (or a *root of multiplicity two*) at ξ:

1. Show that in this case Newton's method is only linear convergent instead of quadratically convergent. *Hint*: study the proof of Newton's method (see first chapter of Süli, linked from the course homepage under the schedule) and use the mean value theorem.
2. Show that if the Newton iteration is replaced with:

$$x_{n+1} = x_n - 2\frac{f(x_n)}{f'(x_n)} \tag{2}$$

quadratic convergence is recovered.