Last updated: Thursday 28th April, 2022 at 16:50.

Written assignment #4

Do only one of the two options for Problem 1.

Problem 1 (option 1). Let $x_0 < x_1$, $f \in C^1([x_0, x_1])$, and let $p \in \mathbb{P}_3$ be a cubic polynomial such that:

$$p^{(i)}(x_j) = f^{(i)}(x_j), \qquad i = 0, 1, \qquad j = 0, 1. \tag{1}$$

Show that p exists and is unique. (*Hint*: for uniqueness, assume that there are two possibilities: $p, q \in \mathbb{P}_3$ such that $p \neq q$. What can you say about the roots of $p - q$?)

Problem 1 (option 2). Prenter: Problem 1, Page 56.

Problem 2. Prenter: Problem 3, Page 56.

Problem 3. Prenter: Problem 4, Page 60.

Problem 4. Consider the quintic Hermite interpolation problem on $[x_0, x_1]$ for the function $f(x) = x^8 - 1$. That is, find $p(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 + \alpha_5 x^5$ such that $p(x_i) = f(x_i)$, $p'(x_i) = f'(x_i)$, and $p''(x_i) = f''(x_i)$ $(i = 0, 1)$:

- Compute $p(x)$ by solving the "Vandermonde system" for this interpolation problem (i.e., solving $\boldsymbol{V}\boldsymbol{\alpha} = \boldsymbol{f}$ for $\boldsymbol{\alpha}$ and write p in the monomial basis).
- Compute $p(x)$ in the cardinal basis. (*Hint*: use the result of part (b) of Problem 3 in this homework).
- Estimate $\max_{x_0 \leq x \leq x_1} |f(x) - p(x)|$. Let $x_0 = 0$ and $x_1 = h$, and rewrite expression in terms of h. Do the same but with $x_0 = 1$ and $x_1 = 1 + h$. As $h \to 0$, which expression goes to zero faster? What can you conclude?
- Let $p(x)$ be the degree 3 Lagrange interpolant for this problem (with uniformly spaced nodes). Solve the previous problem but for this choice of p. What do you observe and what can you conclude?